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Decerns-FT: decision support system for analysis of multi-criteria problems in the fuzzy environment

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Fuzzy multi-criteria decision analysis (FMCDA) is based on assessing functions of fuzzy arguments and ranking of fuzzy numbers. In the general case, implementing each of the above operations requires using the appropriate computer modules. All the current FMCDA systems are based on approximate estimates of the functions of fuzzy arguments. The purpose of this paper is to create and apply the FMCDA system, which implements all the main approaches to evaluating functions of fuzzy numbers as well as different methods for ranking of fuzzy numbers by a fuzzy extension of the classical MCDA method TOPSIS as an example.

The paper presents the functional capabilities of the developed Decerns-FT computer system and its features, including the usability of fuzzy TOPSIS (FTOPSIS) models of various levels of complexity, depending on the chosen method of evaluating functions of fuzzy arguments and the method for ranking fuzzy numbers; it describes the general structure of the system and its major blocks. In this paper, the example of Decerns-FT implementation is presented to analyze distinctions in ranking alternatives within MCDA problems by different FTOPSIS models with the use of approximate methods for estimating functions of fuzzy numbers, standard fuzzy arithmetic, and a reduced and general transformation methods. For this, the Monte Carlo module is used to generate numerous scenarios for multi-criteria problems. Using the Decerns-FT system, it is shown for the first time that distinctions in the ranking alternatives by FTOPSIS models, which differ in approaches to estimating functions of fuzzy numbers and ranking methods, are significant.

The developed computer system Decerns-FT has no analogs in the class of systems that implement FMCDA models. The modules of the Decerns-FT system form the basis for the development of other FMCDA systems, which are components of the DecernsFMCDA decision support system, designed to solve a wide range of scientific and applied problems of multi-criteria decision analysis in conditions of uncertainty/fuzziness, and also for the use within the relevant university courses and training of specialists.

Keywords: *Fuzzy numbers, Ranking of fuzzy numbers, Fuzzy multi-criteria decision analysis, Fuzzy TOPSIS, Fuzzy system, Decerns.*

Multi-criteria decision analysis (MCDA) designed to select "optimal" / compromise solutions (alternatives), their ranking, selection or sorting [1–3]. MCDA is an effective tool for decision support for the selection of reasonable options for action [4–6].

In this paper, a *Fuzzy* MCDA model (FMCDA) refers to a fuzzy extension of an original (classical) MCDA method with the selected implementation of functions of fuzzy numbers (FNs) and a given method for ranking of FNs.

Presently, there are various FMCDA models, including fuzzy extensions of such well-known MCDA methods as TOPSIS, PROMETHEE, and AHP [7, 8].

In this paper, we present a Decerns-FT system that implements a fuzzy extension of the classical TOPSIS method [9]. The first Fuzzy TOPSIS (FTOPSIS) model was created in 1992 [10]. Later, based on the classical TOPSIS method, various FTOPSIS models were developed [7, 11].

At the moment, there are commercial and academic Computer Decision Support Systems (DSSs) that implement MCDA methods. The review [3] considers 47 DSSs implementing such MCDA methods as AHP, ELECTRE, MAVT, MAUT, PROMETHEE, TOPSIS, etc., including DSSs for group decision making [12]. We can also mention the academic system [13], which includes the FMCDA methods FuzzyVIKOR and FuzzyTOPSIS, for which 8 methods of FNs defuzzification are implemented [14]. This system is developed using PHP, MySQL, Ajax, and jQuery. FMCDA models in this system work with triangular and trapezoidal FN, linguistic variables, and group decision analysis are also supported.

At the same time, among the FMCDA models implemented in these systems, including the FTOPSIS models [11], none of the models uses different approaches to estimating functions of FNs and, accordingly, does not investigate the overestimation problem [15], including the problem of implementing dependent FNs in FMCDA models [16–18].

The need to implement and study the FTOPSIS models presented in [17] led to the development of the FuzzyLib library and the Decerns-FT FMCDA system. Exploring the influence of approaches to assessing functions of FNs method and fuzzy ranking methods on the output results (ranks of alternatives) led to the development of a module that allows implementing Monte Carlo methods to get statistical estimates of distinctions for multi-criteria choice (only one, as a rule, "best" alternative is chosen) and ranking [2] (alternatives are ranked from "best" to "worst") problems when using different models of FMCDA.

Fuzzy numbers and methods for ranking of fuzzy numbers

The fuzzy set [19] extends the concept of a classical set, in which the membership function can take values in the segment (closed interval) [0, 1].

One of the key concepts in fuzzy set theory is the concept of an α-*cut* (alpha-cut).

Definition 1. The α-cut Z_α, $\alpha \in (0, 1]$, of a fuzzy set Z with the membership function $\mu_Z(x)$ on a universal set $\mathbb{X}$ is defined as follows: $Z_\alpha = \{x\epsilon\mathbb{X}\colon \mu_Z(x) \geq \alpha\}$.

There are several definitions of FN as a special kind of a fuzzy set with the additional specification of the property of α-cuts or the form of the membership function [15, 20–22]. This paper uses the most general definition of FN [23].

Definition 2. FN Z is a normal bounded fuzzy set in $\mathbb{R}$ in which all the α-cuts Z_α, $\alpha \in (0, 1]$, are segments.

It should be added, fuzzy set/number Z is called normal if its α-cut Z_α for $\alpha = 1$ is not the empty set; FN Z is finite one if its support $supp(Z) = \{x \in \mathbb{R} : \mu_Z(x) > 0\}$ is a bounded set in $\mathbb{R}$; let us also emphasize that a set in $\mathbb{R}$ consisting of a single point, in the context of Definition 2, is also considered as a segment.

It follows from Definition 2 [23] that the closure of the FN Z is a segment, $\overline{supp(Z)} = [c_1, c_2]$, and FN Z can be represented in the following way:

- in the case of $c_1 < c_2$:

$Z = \{(x, \mu_Z(x))\colon \mu_Z(x) > 0 \text{ } if \text{ } x \in (c_1, c_2), \mu_Z(x) = 0 \text{ } if \text{ } x \notin [c_1, c_2]\}$ (1)

- if $c_1 = c_2$, F NZ is a singleton, $Z = c$, with the membership function $\mu_Z(x) = \{1, x = c; 0; x \neq c\}$.

Note also that at points c_1 and c_2 (see expression (1)), the membership function $\mu_Z(x)$, in general, can take values from 0 to 1.

Below, $\mathbb{F}$ denotes the set of FNs according to definition 2.

Taking into account Definition 2 and expression (1), each FN Z is uniquely represented by the set of its α-cut s$Z_\alpha = [A_\alpha, B_\alpha]$, $\alpha \in (0, 1]$, and the segment $[A_0, B_0] = [c_1, c_2]$ (also called, after the above explanations, the α-cut for $\alpha = 0$) [23,24]:

$Z_\alpha = [A_\alpha, B_\alpha]$, $\alpha \in (0, 1]$ (2)

Within FMCDA, so-called triangular and trapezoidal FNs (TrFNs and TpFNs, respectively) play an important role. TrFN Z is characterized by the membership function $\mu_Z(x)$, which has (geometrically) the form of a triangle with vertices (in the plane (x, y)) at points $(a, 0)$, $(b, 1)$, $(c, 0)$, $a \leq b \leq c$ and is denoted by the expression $Z = (a, b, c)$. Similarly, TpFN $Z = (a, b, c, d)$ is defined by four vertices, $(a, 0)$, $(b, 1)$, $(c, 1)$, $(d, 0)$.

Fuzzy preference relations (FPR) play an important role in the comparison and ranking of FNs [23, 25, 26].

Definition 3. A fuzzy preference relation R is a fuzzy relation on $\mathbb{F} \times \mathbb{F}\colon R = ((Z_i, Z_j), \mu_R(Z_i, Z_j))$ in which the membership function $\mu_R(Z_i, Z_j) \in [0, 1]$ represents (given in the framework of R) the degree of preference of Z_i over Z_j.

The separate subclass form so-called reciprocal FPRs with the following property:

$\mu_R(Z_i, Z_j) + \mu_R(Z_j, Z_i) = 1.$ (3)

Definition 4. The ranking of two FNs $Z_i,\ Z_j \in \mathbb{F}$ based on a reciprocal fuzzy preference relation R is defined as follows:

$Z_i \succcurlyeq_R Z_j \text{ } if \text{ } \mu_R(Z_i, Z_j) \geq 0.5;\ Z_i \succ_R Z_j \text{ } if \text{ } \mu_R(Z_i, Z_j) > 0.5;\ Z_i \sim_R Z_j \text{ } if \text{ } \mu_R(Z_i, Z_j) = 0.5\,.$ (4)

Hereinafter, the following denotation will also be used:

$\mu_{ij} = \mu_R(Z_i, Z_j) = \mu_R(Z_i \geq Z_j) = \mu_R(Z_j \leq Z_i).$ (5)

Ranking of FNs represents a key concept in FMCDA. Methods for ranking of FNs can be grouped into three main classes [14,27].

1. *Ranking methods based on defuzzification of FNs.* In these methods, the FNs are represented by the corresponding real numbers with their subsequent ranking [14,24,27-30]. In this paper, we use the following two defuzzification-based FNs ranking methods that are in demand in applications.

Centroid Index (CI) (or Center of Gravity) ranking method.

For FN Z, defuzzification based on the CI method is performed according to the following expression:

$CI(Z) = \frac{\int x\mu_A(x)dx}{\int \mu_A(x)dx}$ (6)

For singleton $Z = c$, $CI(Z) = c$.

Integral of Means (IM) ranking method.

Under this method, defuzzification of FN $Z_\alpha = \{[A_\alpha, B_\alpha]\}$ is based on assessing the following expression:

$IM(Z) = \frac{1}{2}\int_0^1 (A_\alpha + B_\alpha)d\alpha$ (7)

For these methods, ranking of FNs is performed in the following way: the higher the defuzzified value, the higher the rank. The ranking of real numbers is implemented using one of the sorting algorithms. The developed system uses a quick sort algorithm.

2. *Ranking methods based on estimating the distance to the reference fuzzy set.* In these methods, a reference fuzzy set is defined, and for each FN from a given set of FNs, the distance to the reference set is calculated based on the used method. The definition of the reference set is often based on the FNs under consideration [14, 24]. Ranking methods of this class are not used in this paper.

3. *Ranking methods based on the pairwise comparison.* The ordering of a finite set of FNs, in this case, is based on their pairwise comparison. This class is represented by the largest number of FNs ranking methods [27, 30]. In this paper, we use one of the methods for ranking of FNs from this class, based on the Yuan's fuzzy preference relation (Y) [26, 27], which is effectively implemented using alpha-cuts of FN. The Yuan's FPR can be summarized as follows in the form implemented within our research and developments [16–18].

Consider FN $Z_i = \{[A_\alpha^i, B_\alpha^i]\}$, $Z_j = \{[A_\alpha^j, B_\alpha^j]\} \in \mathbb{F}$и$Z_{ij} = Z_i - Z_j = \{[A_\alpha, B_\alpha]\}$. Within the Yuan's FPR $R_Y = ((Z_i, Z_j), \mu_Y(Z_i, Z_j))$, the area S_Y^+ is considered as the "distance" from the positive part of the FN $Z_{ij} = \{[A_\alpha, B_\alpha]\}$ to the axis $\alpha(OY)$, which is defined as follows [16]:

$$S_Y^+(Z_{ij}) = \int_0^1 \big(B_\alpha \theta(B_\alpha) + A_\alpha \theta(A_\alpha)\big) d\alpha, \tag{8}$$

Here $\theta(x)$ is the Heaviside function: $\theta(x) = \{1,\ x \geq 0;\ 0,\ x < 0\}$. The total adjusted area under the membership function of Z_{ij} is estimated by the expression [16, 26]:

$$S_Y(Z_{ij}) = S_Y^+(Z_{ij}) + S_Y^+(Z_{ji}) = \int_0^1 (|B_\alpha| + |A_\alpha|) d\alpha. \tag{9}$$

Definition 5. Let $Z_i, Z_j \in \mathbb{F}$ and $Z_{ij} = Z_i - Z_j$ The Yuan's FPR, $R_Y = ((Z_i, Z_j), \mu_Y(Z_i, Z_j)$, in which $\mu_{ij} = \mu_Y(Z_i, Z_j)$ represents the degree of preference of Z_i over Z_j [26], is defined as

$$\mu_Y(Z_i, Z_j) = S_Y^+(Z_{ij}) / S_Y(Z_{ij})\ \ if\ \ S_Y(Z_{ij}) > 0 \tag{10}$$

and $\mu_Y(Z_i, Z_j) = 0.5\ \ if\ \ S_Y(Z_{ij}) = 0$.

It should be stressed this definition is also valid in the case when Z_i and Z_j are singletons.

The following statements follow directly from definition 5:

$$\mu_Y(Z_i, Z_j) = \mu_Y(Z_i - Z_j, 0);\ Z_1 \succcurlyeq_Y Z_2\ iff\ (Z_1 - Z_2) \succcurlyeq_Y 0. \tag{11}$$

The Yuan's FPR is reciprocal and transitive [26].

To rank FNs using Yuan's FPR, a quick sort algorithm is used, where comparisons of FNs (as for real numbers) is based on calculating the difference between two quantities (based on relations (4)).

The Decerns-FT system implements also ranking of FNs based on the FRAA (Fuzzy Rank Acceptability Analysis) concept [16,31] using (within the FRAA framework) the Yuan's FPR. It is proved [16], Yuan's and $FRAA_Y$ ranking are equivalent. However, FRAA ranking of FNs (alternatives within an FMCDA problem) allows for each pair r and k ($1 \leq r$, $k \leq n$, where n is the number of considered FNs/alternatives) to determine the fuzzy measure (confidence level, Fuzzy Rank Acceptability Index) that FN/alternative k has the rank r.

One of the key concepts of the FMCDA is the implementation of functions of FNs.

Let $G \subseteq \mathbb{R}^n, f: G \to \mathbb{R}$ is a real function. The extension of the function $= f(x_1, \ldots, x_n)$ to the function $= f(Z_1, \ldots, Z_n)$ of fuzzy arguments, when FN Z_i is used instead of real numbers x_i, is defined on the basis of Zadeh's extension principle [21,24,32]: membership function of FN $Z = f(Z_1, \ldots, Z_n)$ is defined by the expression:

$$\mu_Z(z) = \vee_{z = f(x_1, \ldots, x_n)} (\wedge_{i=1,\ldots,n} \mu_{Z_i}(x_i)), \tag{12}$$

$z \in \mathbb{R}$; and $\mu_Z(z) = 0$ if the preimage of the point z is empty: $f^{-1}(z) = \emptyset$ (here $a \wedge b = \min(a, b)$, $a \vee b = \max(a, b)$; $a, b \in \mathbb{R}$). However, direct application of the expansion principle based on expression (12) is inefficient even for simple functions. To calculate a function of FNs, we use approximate calculations, standard fuzzy arithmetic (SFA), and transformation methods (TMs) [15].

Below, we give an example of the *overestimation* problem that occurs when using SFA (including approximate calculations that represent a narrowing of SFA to its use on the α-cut $\alpha = 1$ and, for $\alpha = 0$, on the segment $[A_0, B_0]$, see the representation of FN (2)). Consider the positive FNs A, B and functions

$$Z_O = (A + B)/A, \quad Z_T = 1 + B/A \tag{13}$$

that are fuzzy extensions of real functions

$$f_1(a, b) = \frac{a+b}{a}, \quad f_2(a, b) = 1 + b/a \tag{14}$$

Obviously, for the real numbers $a, b, a \neq 0, f_1(a, b) \equiv f_2(a, b)$. At the same time, using SFA, $supp(Z_T) \subset supp(Z_0)$, Figure 1.

The overestimation problem arises when there are dependent variables in the fuzzy expression under consideration [15,18]: in (13) for the case of Z_0, the dependent variables are the numerator, $A + B$, and the denominator, A. At the same time, f_1 and f_2 (13) are equivalent, so the estimates of Z_0 and Z_T based on the extension principle (the proper estimate of the function of FNs) are also equivalent and, in the cases under consideration, coincide with Z_T. We add also that the use of TM to assess Z_0, as well as the use of SFA to calculate Z_T, leads to the proper evaluation of these expressions.

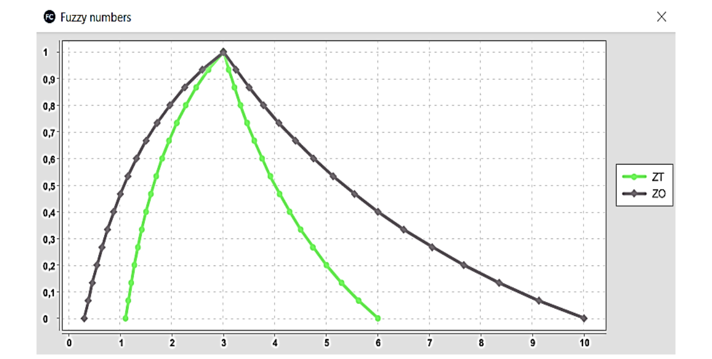

Fig. 1. An example of the overestimation (SFA is implemented): FN Z_0 and Z_T (13); A, B are TrFNs: A = (1, 2, 5), B = (0.5, 4, 5)

To overcome the problem of dependent variables when calculating the proper result of a function $f(Z_1, \ldots, Z_n)$, as well as to calculate the output value for non-monotonic real functions $f(x_1, \ldots, x_n)$, transformation methods (TMs) can be used, which are one of the variants of interval computing on α-cuts. TMs are described in detail in [15].

Let's consider a real continuous function $f(x_1, \ldots, x_n)$, $f: \mathbb{R}^n \to \mathbb{R}$, and its fuzzy extension, i.e., the function $f(Z_1, \ldots, Z_n)$, where Z_i, $i = 1, \ldots, n$, are FNs.

Implementing the main options of TMs can be represented as follows.

1. If the real function $f(x_1, \ldots, x_n)$ is monotone for every x_i, $i = 1, \ldots, n$, in the segment $U_i = [A_0^i,\ B_0^i]$, i.e., for differentiable functions, $\partial f/\partial x_i$ does not change its sign in the segment U_i (for fixed values of other variables in the corresponding segments $U_j, j \neq i$), the Reduced Transformation Method (RTM) is used: for each $\alpha \in [0, 1]$, segments $Z_a^i = [A_a^i, B_a^i]$are considered and the values of $Y = f(X_1, \ldots, X_n)$ are calculated for all combinations of $\{X_1, \ldots, X_n\}$, where X_i is one of the boundary points of the segment Z_a^i, i.e. $X_i \in \{A_a^i, B_a^i\}$, with subsequent estimation of the minimum and maximum values of Y for the formation of α-cut $Z_\alpha = [A_\alpha, B_\alpha]$ of the estimated FN $Z = f(Z_1, \ldots, Z_n)$.

2. If the function $f(x_1, \ldots, x_n)$ is not monotonic for everyx_i in the segment U_i, the General Transformation Method (GTM) is applied: for each α-cut, values $Y = f(X_1, \ldots, X_n)$ are calculated for all combinations $\{x_1, \ldots, x_n\}$, where X_i is one of the N_α points in the segment $[A_a^i, B_a^i]$, for this segment $[A_a^i, B_a^i]$is divided into $N_\alpha - 1$ intervals (according to a special algorithm) by points $C_1, \ldots, C_{N_\alpha - 2}$, i.e., $X_i \in \{A_a^i, C_1, \ldots, C_{N_a-2}, B_a^i\}$; then minimum and maximum of the calculated values Y are taken to form the α-cut $Z_\alpha = [A_\alpha, B_\alpha]$.

We add that RTM can also be used in this case as a simplified approach; at the same time, it is necessary to increase the number of used α-cuts.

3. In the general case, the function $f(x_1, \ldots, x_n)$ can be monotonic regarding the variables x_i in the segments U_i, $i = 1, \ldots n_1$, and non-monotonic regarding the other variables in the corresponding segments. Here, to reduce the time of calculation, instead of GTM, Extended TM (ETM) is used: for "monotone variables" x_i, RTM is implemented, for the remaining variables, GTM is used.

Despite some approximation, we will call the output value Z, obtained based on a correct application of TM, the proper value for the function of FNs.

According to the algorithms RTM and GTM, the amount of the operations when calculating FN $Z = f(Z_1, \ldots, Z_n)$ and, consequently, running time of algorithms can be represented as $O(N_\alpha k^n)$, where k is the maximum number of points used in α-cuts, N_α is the number of α-cuts; within RTM, $k = 2$. To significantly reduce the computing time for implementing FMCDA models, including one within the *Decerns - FT* system, the process of parallel calculations is implemented.

FTOPSIS models

This section describes various models of FTOPSIS as a fuzzy extension of the classical TOPSIS method.

The FTOPSIS model implies the implementation of the following basic steps [17,18].

Step 1. Defining alternatives A_j, $i = 1, 2, \ldots,$ and criteria C_j, $j = 1, 2, \ldots, m$; forming a performance table $\{C_{ij}\}$, where C_{ij} is a (fuzzy) value of the criterion C_i for alternative A_i. As a rule, within FMCDA problems, TrFNs and TpFNs are used. It is also possible to use linguistic variables (LV) and FNs of a general type; $C_{ij} = \{[A_a^{ij}, B_a^{ij}]\}$; w_j - (fuzzy) weight coefficient of the criterion C_j, $w_j = \{[A_a^j, B_a^j]\}$.

Step 2. Normalization of C_{ij} values. For criteria values, $C_{ij} = \{[A_a^{ij}, B_a^{ij}]\}$, we evaluate the following boundary values:

$$B_0^{0j} = \max_{i=1,\ldots,n} B_0^{ij}, \quad A_0^{0j} = \min_{i=1,\ldots n} A_0^{ij}. \tag{15}$$

Normalization is the conversion of criteria values to dimensionless scale: $C_{ij} \to x_{ij}$. For benefit criteria (the more, the better), it applies the following normalization procedure:

$$x_{ij} = (C_{ij} - A_0^{0j})/(B_0^{0j} - A_0^{0j}); \tag{16}$$

for cost criteria (the less, the better):

$$x_{ij} = (B_{ij} - C_0^{0j})/(B_0^{0j} - A_0^{0j}), \tag{17}$$

Thus, for FNx_{ij}, $\overline{supp}(x_{ij}) \subseteq [0,1]$, and in the dimensionless x-scale, all criteria are positive.

Step 3. Choose the "ideal" and "anti-ideal" alternatives. The "ideal", I^+, and "anti-ideal", I^-, alternatives in *m*-dimensional space (*m* is the number of criteria) are defined as follows:

$$I^+ = (1, \dots, 1), \quad I^- = (0, \dots, 0). \tag{18}$$

The choice of these (global) "ideal" and "anti-ideal" alternatives [17] greatly simplifies the process of assessing functions of FNs within FTOPSIS.

Step 4. Determine the distances for each alternative A_i from the "ideal", D_i^+, and" anti-ideal", D_i^-, alternatives. In the normalized criteria space, the weighted distance from the alternative $x_i = (x_{i1}, \dots, x_{im})$ (with fuzzy values x_{ij}) to the "ideal", I^+, and "anti-ideal", I^-, alternatives is determined by the following expression:

$$D_i^+ = d(x_i, I^+) = \left(\sum w_k^p (x_{ik} - I_k^+)^p\right)^{1/p} = \left(\sum w_k^p (1 - x_{jk})^p\right)^{1/p}, \tag{19}$$

$$D_i^- = d(x_i, I^-) = \left(\sum w_k^p (x_{ik} - I_k^-)^p\right)^{1/p} = \left(\sum w_k^p x_{ik}^p)^p\right)^{1/p}, \tag{20}$$

where D_i^+ and D_i^- are FNs. In this paper, we consider the classical case with the value $p = 2$.

Step 5. The generalized criterion (coefficient of closeness) Z_i for the alternative A_i is determined by the expression

$$D_i = D_i^- / (D_i^- + D_i^+), \tag{21}$$

$$D_i = \left(\sum w_k^p x_{ik}^p\right)^{1/p} / \left(\left(\sum w_k^p x_{ik}^p\right)^{1/p}\right) + \left(\sum w_k^p (1 - x_{ik})^p\right)^{1/p}\right), \tag{22}$$

FNs w_k and x_{ik} occur in the numerator and in the denominator, which leads to the overestimation when using SFA. To determine the proper value of D_i (22), the RTM can be used.

Step 6. Ranking of alternatives is based on ranking of FNs D_i, $i = 1, \dots, n$ (22).

Depending on the methods used for estimating functions of FNs and ranking FNs, the following models can be used.

FTTr model (Fuzzy TOPSIS with Triangular Fuzzy Numbers, TrFNs), where approximate calculations are used to get a generalized criterion D_i and the difference $D_{ij} = D_i - D_j$. The FTS (Fuzzy TOPSIS with Standard Fuzzy Arithmetic) model, where standard fuzzy arithmetic is used to get the generalized criterion D_i and the difference $D_{ij} = D_i - D_j$. The FTR model (Fuzzy TOPSIS with Reduced Transformation Method, RTM), where RTM is used to assess generalized criterion D_i(expression (22)) and the difference $D_{ij} = D_i - D_j$.

Depending on the used ranking method, there are the FTTrCI and FTTrIM models, where the CI and IM methods are used to rank alternatives, respectively; the FTSCI, FTSIM, FTRCI, FTRIM models, and the FTRY model, where the Yuan's (Y) ranking method is used. It is worth noting that the implementation of the FTRY model, because of the use of TM for calculations requires much more time in comparison with other models.

Computer System Decerns-FT

This section describes the structure of Decerns-FT system, and the application of the system to investigate the distinctions of FTOPSIS models described in earlier.

The structure of Decerns-FT

To implement FTOPSIS fuzzy models used in the Decerns-FT system, and the module for comparing fuzzy models based on Monte Carlo simulation, the FuzzyLib library was developed. It implements the library in the Java programming language. Figures 2 and 3 show the UML class diagram of the developed FuzzyLib library.

The "Fuzzy Math and Ranking" category includes classes that implement the process of calculating functions of FNs and FNs ranking.

1. The PRanking and DRanking classes implement ranking methods based on pairwise comparisons and defuzzification, respectively. The PRankingclass includes the pairwiseCompare method, which is designed for pairwise comparison of FNs. As arguments, it takes an array of fuzzy numbers fuzzySets and an object of the PairwiseComparison class, which implements the comparison of a pair of FNs. The FRAA method [16] implements the FRAA procedure for ranking a set of FNs based on the pairwiseCompareTable, which outputs an array of generated objects of the PRank class. The object of the PRank class contains a field denoting the rank defined by the FRAA method and the FRAI rankmembershipvalue array. The DRanking class includes a ranking method designed to rank of FN fuzzySets based on the defuzzification method of the Defuzzification class object (dm argument). The output is an array of objects of the DRank class, which contains the rank number, the value of the dValue defuzzification method, and FN of fSet.

2. The fuzzymath class. This class implements methods for evaluating functions of FNs. As input arguments, it takes an expression built based on objects of the Node class in the tree type.

Each descendant of the Node class implements the evaluateTM and evaluateAC methods to calculate the input values individually (which is required in the RTM and GTM methods) or alpha-cuts as a whole (approximate calculation methods and standard fuzzy arithmetic). Objects of the VariableNode class using fuzzySetName allow to get an alpha-cut or the FN value considered by the transformation method from the fuzzySetValues argument.

The Utility category includes classes describing the FNs (FuzzySet and its descendants), a mathematical function class (this class is required for obtaining alpha-cuts of a FN and evaluating integrals in the CI, IM,

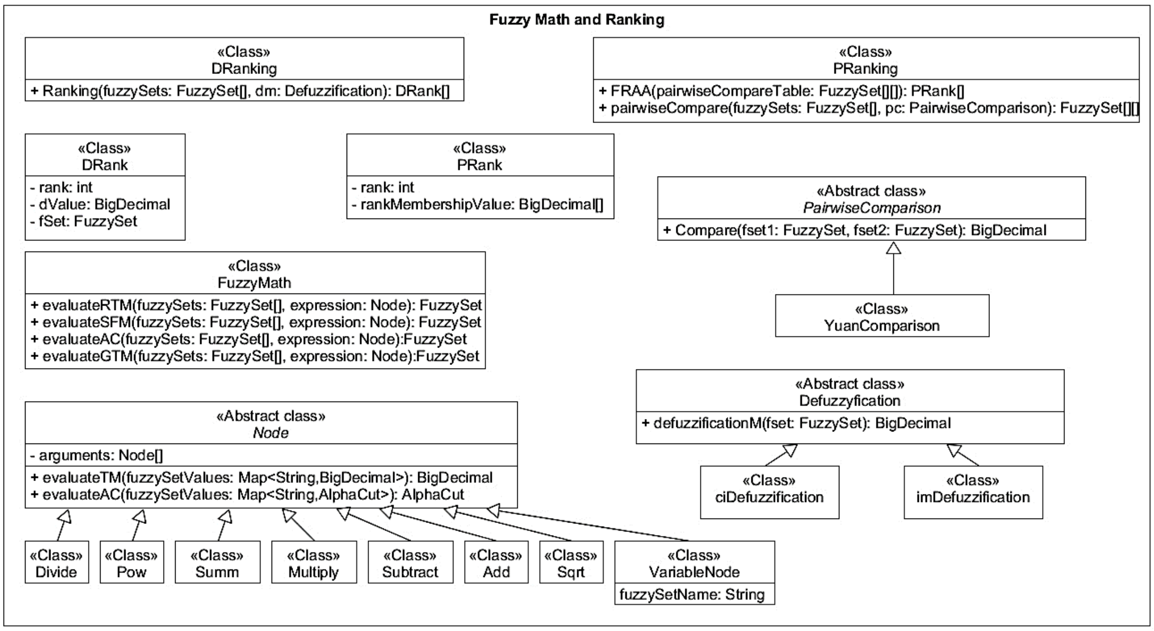

Fig. 2. Fuzzy Math and Ranking Class category of the FuzzyLib library

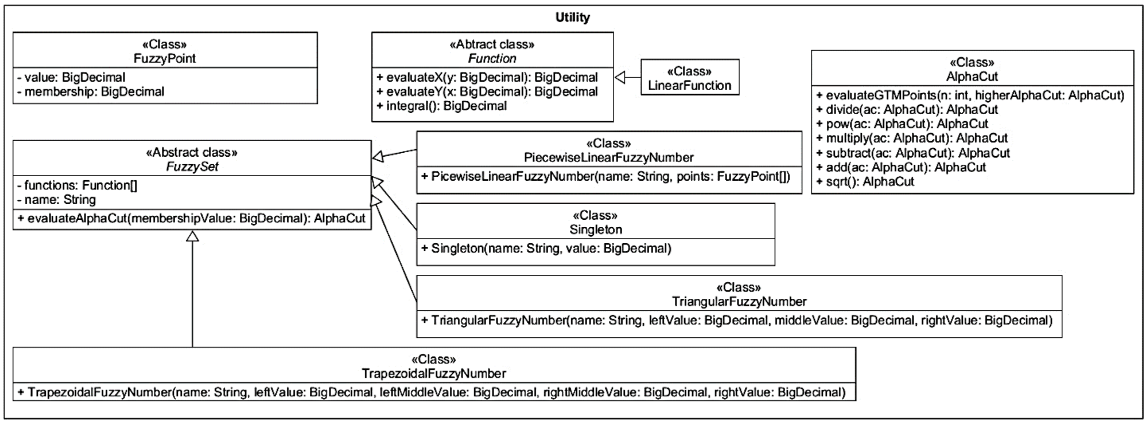

Fig. 3. Utility class category of the FuzzyLib library

and Y (Yuan's) ranking methods, and some other classes (for example, the AlphaCut class, which includes a method for determining auxiliary points in the GTM method and methods for evaluating basic mathematical operations, including addition, subtraction, division, multiplication, etc., using approximate calculations and standard fuzzy arithmetic).

The conceptual scheme of the Decerns-FT system, and the main modules of the FuzzyLib library, are presented in Figures 4 and 5, respectively.

The graphical interface of Decerns-FT includes modules for implementing the next steps in solving an FMCDA problem [17].

1. The value tree formation [33].
2. In the feature table, fuzzy values of criteria for the considered alternatives are set; the types of FNs membership functions used in Decerns-FT are shown in Figure 6, respectively: singletons, triangular, trapezoidal, piecewise linear (continuous, upper continuous), Gaussian, piecewise linear upper semi-continuous, bell-shaped, and FNs with sigmoid membership functions.
3. Setting fuzzy weight coefficients using the FSwing method [31] or the direct waiting method. Figure 7 shows an example of setting weights using FSwing.
4. Applying FTOPSIS models to solve a multi-criteria problem. The Decerns-FT system includes fuzzy models described in Section 3, see also Figure 4.

The adapted version of the desktop Decerns-FT system is also a subsystem of the modified and extended version of the DecernsMCDA [33, 34], which includes the following classical MCDA methods: MAVT, TOPSIS, AHP, PROMETHEE, MAUT, ProMAA, and FlowSort.

Decerns-FT	
Tools	**Fuzzymodels**
Value tree Performance table Pareto dominance	1. FTTrCl 2. FTTrIM 3. FTSCl 4. FTSIM 5. FTRCl 6. FTRIM 7. FTRY
Types of membership functions for FNs	**Methods for setting weight coefficients**
Triangular FNs Trapezoidal FNs Piecewise linear FNs Gaussian, bell-shaped, sigmoid FNs	The direct method FSwing method

Fig. 4. Conceptual diagram of the Decerns-FT system

FuzziLib		
Methods for ranking FNs	**Fuzzy models FTOPSIS**	**Method for estimating a function of FNs**
Ranking based on defuzzification	1. FTTrCl 2. FTTrIM 3. FTSCl 4. FTSIM 5. FTRCl 6. FTRIM 7. FTRY	**Approximate calculations**
1. Centroid Index (CI) 2. Integral of Means (IM) 3. Median method		**Standard Fuzzy Arithmetic**
Ranking based on pairwise comparisons		**Transformation methods (RTM, GTM)**
1. The Yuan's method (Y) 2. FRAA method		
	Monte Carlo algorithms	

Fig. 5. FuzzyLib library

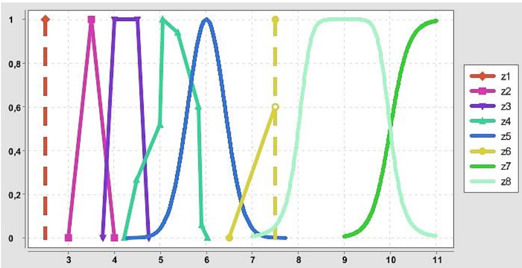

Fig. 6. Types of fuzzy numbers used in Decerns-FT

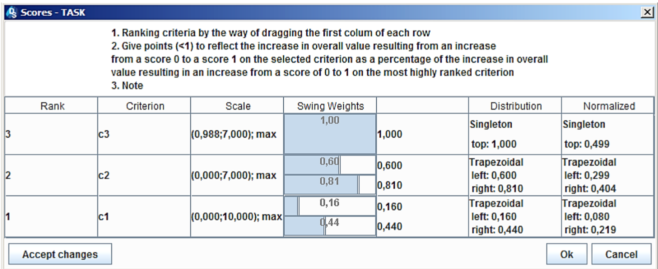

Fig. 7. Setting weights by FSwing in Decerns-FT

Comparison of FTOPSIS models using the Monte Carlo simulation

Consider the module for comparing FTOPSIS models implemented in Decerns-FT using Monte Carlo algorithms. The comparison is made by the number of distinctions in ranks of alternatives formed based on Monte Carlo simulation within the generated scenarios of fuzzy multi-criteria problems.

In this paper, there is a comparison of the ranks of alternatives according to FTRY model with the ranks by FTTrIM, FTSIM, FTRCI, and FTRIM models, Tables 1–3.

The paper considers the FMCDA problems with $m = 4$ criteria $n = 4$ alternatives. Based on Monte Carlo algorithms, 5000 iterations are generated, each of which generates a multi-criteria problem scenario. At the same time, we study both the multi-criteria choice problem (only the rank $r_1 = 1$ is considered) and the ranking problem (all ranks $r_2 = \{1, \ldots, 4\}$ are taken into account). Within the scenario, using TrFNs in the segment [0, 1], a performance table is generated (the values of alternatives according to criteria) along with weight coefficients using a uniform distribution in [0, 1].In this paper, three variants of TrFNs are generated: symmetric TrFNs ($v = 1$), general/non-symmetric TrFNs ($v = 2$), and terms of the linguistic variable (based on the seven-term scale of TrFNs in [0, 1]) ($v = 3$) [23]. For each scenario ($t = 1, \ldots, N_{\max} = 5000$) and variant/form of TrFNs ($v = 1, 2, 3$) and for the specified fuzzy model M_k, the ranks of alternatives were evaluated and compared with the ranks of the basic model M_0 (FTRY).Number of distinctions, $D(t + 1; M_k, r_l, v)$, $t = 0, 1, \ldots, N_{\max} - 1$; $k = 1, \ldots, 4$; $l = 1, 2$; $v = 1, 2, 3$, at each iteration for the compared pair increased by one if a distinction was found in at least one of the ranks of the sets r_1 or, respectively, r_2: $D(t + 1; M_k, r_l, v) +1$, if there was no distinction in rank at iteration $t + 1$, then $D(t + 1; M_k, r_l, v) = D(t; M_k, r_l, v)$. The frequency of distinctions in the ranks of alternatives when using different FTOPSIS methods (statistical assessment of the probability of distinction) was defined as $p = D(N_{\max}; M_k, r_l, v)/N_{\max}$; $D(0; M_k, r_l, v) = 0$. Note that estimates of distinctions were also made for the intermediate number of iterations $N_{\max} = 1000$; the latter indicates the level of distinctions in the ranks of alternatives with an increase in the (maximum) number of iterations. A detailed description of the algorithm for assessing distinctions and corresponding features is described, for example, in [35–37]. The frequency of distinctions depends on the dimension of the MCDA problem (i.e., on the number of criteria and alternatives); if the dimension of the MCDA problems increases, the percentage of differences also increases [35–37]. In the developed module (in the settings), the number of α-cutsN_α to analyze the degree of influence of the number of α-cuts on the output results can be specified; in Tables 1–3, N_α = 15.

Table 1

Relative frequency of distinctions (%) in ranking alternatives between FTTrIM, FTSIM, FTRCI, FTRIM and the FTRY base model for ranks 1/(1–4). Criteria values and weigh coefficients are asymmetric TrFNs

Number of iterations	FTTrIM	FTSIM	FTRCI	FTRIM
N1 =1000	70.8/90.2	58/83.1	10/26.4	2.4/8.7
N2 =5000	68.5/89.3	57.2/82.6	9/26.1	2.9/8.9

Table 2

Relative frequency of distinctions (%) in ranking alternatives between FTTrIM, FTSIM, FTRCI, FTRIM and the FTRY base model for ranks 1/(1–4). Criteria values and weight coefficients are symmetric TrFNs

Number of iterations	FTTrIM	FTSIM	FTRCI	FTRIM
N1 =1000	72.4/89.7	63.1/84.5	3.4/11	2.1/8.3
N2 =5000	61.4/90.2	62.7/84.4	3.44/12.1	2.5/9.3

Table 3

Relative frequency of distinctions (%) in ranking alternatives between FTTrIM, FTSIM, FTRCI, FTRIM and FTRY base model for ranks 1/(1–4). Criteria values and weight coefficients are terms (TrFNs) of the linguistic variable

Number of iterations	FTTrIM	FTSIM	FTRCI	FTRIM
N1 =1000	7.7/21.4	4.6/15	0.7/3.3	0.4/2.2
N2 =5000	8.5/22.7	5.4/15.3	1.1/3.7	0.4/1.8

Analysis of distinctions in ranking of alternatives by FTOPSIS models (Tables 1, 2) shows that the influence of the shape of the input FNs used (symmetric/asymmetric TrFNs) is insignificant; the exception is the distinctions between the basic model and the FTRCI model for asymmetric TrFNs (in contrast to the FTRIM model, which differs from the previous one only by the ranking method). According to the estimates, distinctions in ranks of alternatives for FTOPSIS models are significant both for ranking problems (82–90 %) and for the choice problem (57–68 %). With increasing the precision for assessing functions of FNs, from FTTrIM as the least accurate (based on approximate calculations based on propagation of TrFNs through the whole chain of calculations), to FTSIM as more accurate model (with the use of SFA, but not consider the dependence of FNs, in contrast to models with the use of TM: model FTRY and models FTRCI and FTRIM) distinctions are reduced. Distinctions in ranking when comparing the basic model with the "proper" models, FTRCI and FTRIM, can be considered, when using as input symmetric FNs (such FNs, as a rule, are used in applied problems), as small/insignificant (about 3.5 %) for problems of multi-criteria choice and "conditionally acceptable" (9–12 %) for problems of ranking alternatives.

The least of differences in ranks of alternatives for these FTOPSIS models were noted when using linguistic variables (which is very popular in the framework of FMCDA [7, 30]) (Table 3). For models with an approximate approach to the calculating functions of FNs (FTTrIM and FTSIM), we can consider the distinctions for the choice problems as acceptable (5–8.5 %), while for the ranking problems (15–23 %), we can characterize the distinctions as significant. For models with the "proper" calculations, FTRCI and FTRIM, the distinctions in ranks are quite small, both for choice problems (0.4–1.1 %) and for ranking problems (1.8–3.7 %), which in terms of uncertainty/fuzziness is quite an acceptable discrepancy in the framework of decision analysis problems.

Thus, the developed Decerns-FT system makes it possible not only to solve applied multi-criteria problems [17] but also to conduct scientific research within FMCDA [18, 36].

Conclusion

An analysis of existing publications in the field of fuzzy MCDA (FMCDA) shows that all the known papers use approximate methods for estimating functions of fuzzy numbers and, as a rule, one method for ranking fuzzy numbers based on the use of the centroid index (CI).

The paper presents the original Decerns-FT system for solving practical [17] and research [18, 36] problems of multi-criteria decision analysis (MCDA) under conditions of uncertainty/fuzziness. The system implements various methods for evaluating functions of fuzzy arguments and several methods for ranking of fuzzy numbers. The developed Decerns-FT system allowed us to show for the first time [36] that estimates of the ranks of alternatives based on approximate FMCDA models can significantly differ from the corresponding estimates with the use of more accurate models, which implement the transformation method.

The developed library of modules allows using the system for solving a wide range of applied and research problems of FMCDA. The adapted version of the desktop Decerns-FT system is also one of the subsystems of the fuzzy decision support system DecernsFMCDA under creation, which is a further development of the DecernsMCDA system [33] for solving MCDA problems. Algorithms and modules implemented in Decerns-FT are also used in the development of systems (as components of the DecernsFMCDA integrated system), which are fuzzy extensions of such MCDA methods as PROMETHEE, MAVT, FlowSort, and others.

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Система поддержки принятия решений Decerns-FT для анализа многокритериальных задач в условиях нечеткости

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В основе нечеткого многокритериального анализа решений (НМКАР) лежат операции оценки функций от нечетких аргументов и ранжирования нечетких чисел. В общем случае реализация каждой из указанных операций требует использования соответствующих компьютерных модулей. Все известные на настоящее время системы НМКАР базируются на приближенных оценках функций от нечетких аргументов. Целью представленной работы является создание и научно-практическое применение системы НМКАР,

реализующей все основные подходы к оценке функций от нечетких чисел, а также различные методы ранжирования нечетких чисел на примере нечеткого расширения классического метода многокритериального анализа решения TOPSIS.

В работе представлены функциональные возможности разработанной компьютерной системы Decerns-FT и ее особенности, включающие возможность использования нечетких моделей FTOPSIS различного уровня сложности в зависимости от выбранного метода оценки функций от нечетких аргументов и метода ранжирования нечетких чисел. Описана общая структура системы и ее основные блоки. Приведен пример использования Decerns-FT для анализа различий в ранжировании альтернатив многокритериальных задач разработанными моделями FTOPSIS с применением приближенных методов оценки функций от нечетких чисел, методов стандартной нечеткой арифметики, а также редуцированного и общего метода трансформации. В рамках решения данной задачи используется модуль Монте-Карло для генерации большого числа сценариев многокритериальных задач. С использованием системы Decerns-FT впервые показано, что различия в ранжировании альтернатив многокритериальных задач моделями FTOPSIS, отличающимися подходами к оценке функций от нечетких чисел и методами ранжирования, являются значимыми.

Разработанная компьютерная система Decerns-FT не имеет аналогов в классе систем, реализующих модели НМКАР. Модули системы Decerns-FT формируют основу для создания других систем НМКАР, являющихся компонентами системы поддержки принятия решений DecernsFMCDA, предназначенной для решения широкого круга научно-прикладных задач многокритериального анализа решений в условиях неопределенности/нечеткости, а также для использования в рамках соответствующих университетских курсов и для тренинга специалистов.

Ключевые слова: *нечеткие числа, ранжирование нечетких чисел, нечеткий многокритериальный анализ решений, Fuzzy TOPSIS, нечеткая система, Decerns.*

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